

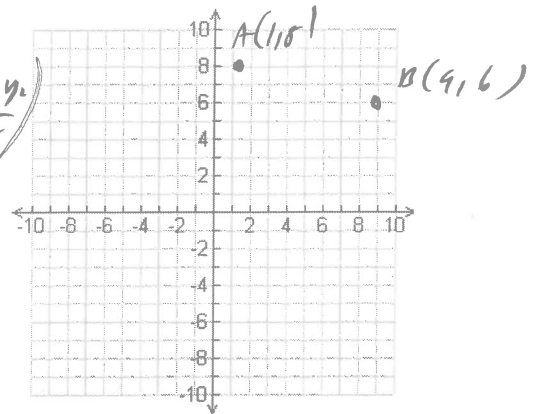
Midpoint and Distance and Slope 6.1 6.2 Worksheet

$$\text{midpt} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \text{Part 1: Graphing} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1) Graph the points A (1, 8) and B (9, 6). Find the midpoint of \overline{AB} . Write as an exact value. Find the length of \overline{AB} .

$$\begin{aligned} d_{AB} &= \sqrt{(1-9)^2 + (8-6)^2} \\ &= \sqrt{(-8)^2 + (2)^2} \\ &= \sqrt{64+4} \\ &= \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$$

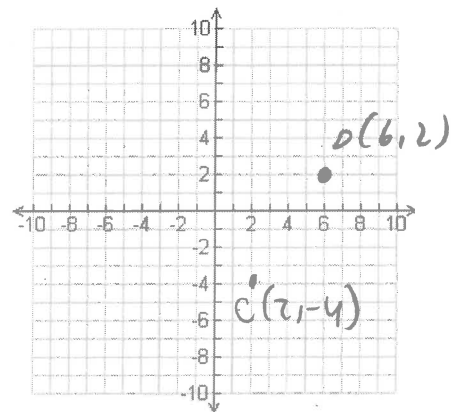
$$\begin{aligned} \text{midpt } \overline{AB} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{1+9}{2}, \frac{8+6}{2} \right) \\ &= \left(\frac{10}{2}, \frac{14}{2} \right) \\ &= (5, 7) \end{aligned}$$



2) Graph the points C (2, -4) and D (6, 2). Find the midpoint of \overline{CD} . Find the length of \overline{CD} .

$$\begin{aligned} d_{CD} &= \sqrt{(2-6)^2 + (-4-2)^2} \\ &= \sqrt{(-4)^2 + (-6)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$$

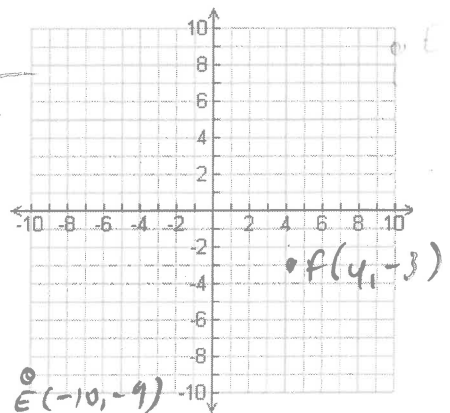
$$\begin{aligned} \text{midpt } \overline{CD} &= \left(\frac{2+6}{2}, \frac{-4+2}{2} \right) \\ &= \left(\frac{8}{2}, \frac{-2}{2} \right) \\ &= (4, -1) \end{aligned}$$



3) Graph the points E (-10, -9) and F (4, -3). Find the midpoint of \overline{EF} . Find the length of \overline{EF} .

$$\begin{aligned} \text{midpt } \overline{EF} &= \left(\frac{-10+4}{2}, \frac{-9-3}{2} \right) \\ &= \left(\frac{-6}{2}, \frac{-12}{2} \right) \\ &= (-3, -6) \end{aligned}$$

$$\begin{aligned} d_{EF} &= \sqrt{(-10-4)^2 + (-9-3)^2} \\ &= \sqrt{(-14)^2 + (-12)^2} \\ &= \sqrt{196+144} \\ &= \sqrt{340} \\ &= 2\sqrt{85} \end{aligned}$$



$$\text{midpt} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Part 2: Midpoint Using Formula Only

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the midpoint for each line segment using the formula (no graphing needed). Show the formula and all work.

4) G (6, 5) and H (9, 2)

$$\begin{aligned} \text{midpt } \overline{GH} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{6 + 9}{2}, \frac{5 + 2}{2} \right) \\ &= \left(\frac{15}{2}, \frac{7}{2} \right) \end{aligned}$$

5) I (1, 1) and J (-3, -3)

$$\begin{aligned} \text{midpt } \overline{IJ} &= \left(\frac{1 + (-3)}{2}, \frac{1 + (-3)}{2} \right) \\ &= \left(\frac{-2}{2}, \frac{-2}{2} \right) \\ &= (-1, -1) \end{aligned}$$

6) Given the midpoint of segment KL is M (1, -1) and L (8, -7). What are the coordinates of the other endpoint K? (x, y)

$$\text{midpt } \overline{KL} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(1, -1) = \left(\frac{x + 8}{2}, \frac{y - 7}{2} \right)$$

$$1 = \frac{x + 8}{2}$$

$$2 = x + 8$$

$$-6 = x$$

Point
(-6, 5)

Part 3: Distance Using Formula Only

$$-1 = \frac{y - 7}{2}$$

$$-2 = y - 7$$

$$5 = y$$

Find the distance between each set of points. Show the formula and all work.

7) (0, 0) and (4, 3)

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - 4)^2 + (0 - 3)^2} \\ &= \sqrt{(-4)^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

8) (3, -3) and (2, 7)

$$\begin{aligned} d &= \sqrt{(3 - 2)^2 + (-3 - 7)^2} \\ &= \sqrt{1^2 + (-10)^2} \\ &= \sqrt{1 + 100} \\ &= \sqrt{101} \end{aligned}$$

9) Determine the coordinates of the points needed. Then find the distance of each line segment.

a) GH

G (-8, 6)

H (1, 8)

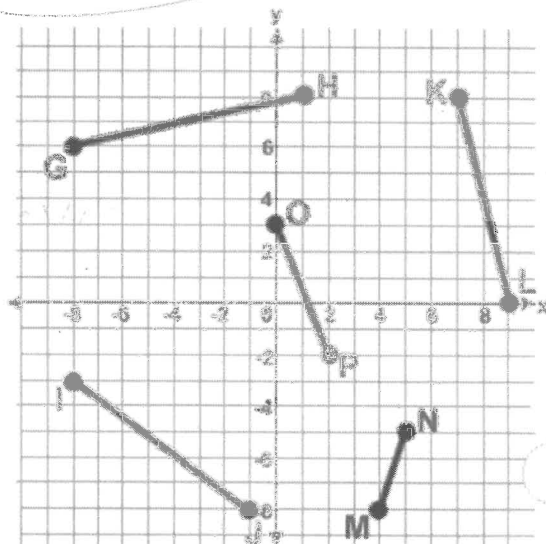
$$\begin{aligned} d &= \sqrt{(-8 - 1)^2 + (6 - 8)^2} \\ &= \sqrt{(-9)^2 + (-2)^2} \\ &= \sqrt{81 + 4} = \sqrt{85} \end{aligned}$$

b) KL

K (9, 8)

L (8, 0)

$$\begin{aligned} d &= \sqrt{(9 - 8)^2 + (8 - 0)^2} \\ &= \sqrt{1^2 + 8^2} \\ &= \sqrt{65} \end{aligned}$$



Part 4: Putting it All Together

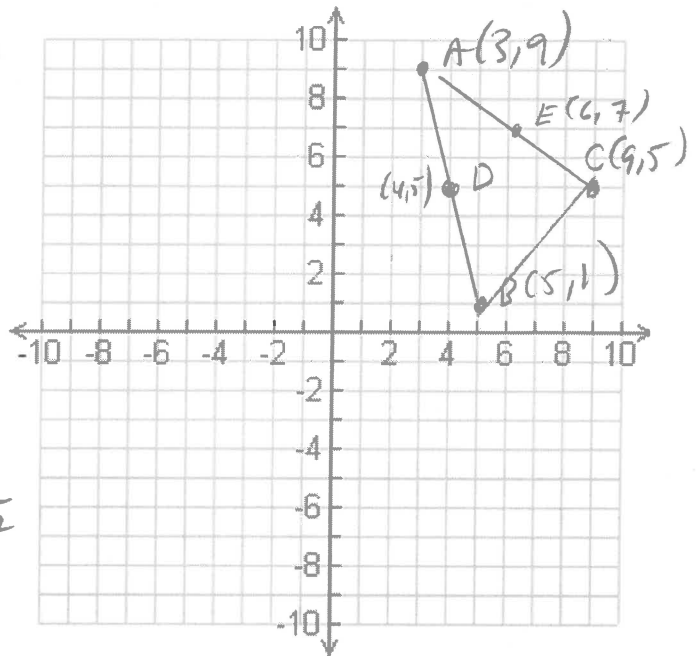
10) Triangle ABC has coordinates A (3, 9), B (5, 1) and C (9, 5). D is the midpoint of AB and E is the midpoint of AC.

- a) Graph the points A, B, and C (make sure you label them). Find the coordinates of points D and E. Show all work.

$$\left(\frac{3+5}{2}, \frac{9+1}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right) = (4, 5)$$

D =

$$E = \left(\frac{3+9}{2}, \frac{9+5}{2}\right) = \left(\frac{12}{2}, \frac{14}{2}\right) = (6, 7)$$



- b) Plot points D and point E on the graph and label.

- c) Find the length of BC. Show all work.

$$\sqrt{(5-9)^2 + (1-5)^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$$

- d) Find the length of DE. Show all work.

$$\sqrt{(4-6)^2 + (5-7)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

Problems

- 11) Find x and y if (2, 5) is the midpoint of points (x, y) and (-5, 6).

$$(2, 5) = \left(\frac{x-5}{2}, \frac{y+6}{2}\right) \quad 2 = \frac{x-5}{2} \quad \frac{y+6}{2} = 5$$

$$4 = x - 5 \quad 9 = x$$

$$10 = y + 6 \quad y = 4$$

$$(9, 4)$$

$$\angle BAC = 90^\circ \quad \overline{AB} = \overline{AC}$$

- 12) Show that the triangle that has (0, 1), (2, 3) and (2, -1) as vertices is right isosceles.

$$\overline{AB} = \overline{AC}$$

$$\begin{aligned} \overline{AC} &= \sqrt{(0-2)^2 + (1+1)^2} \\ &= \sqrt{(-2)^2 + (2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$



$$m_{AB} = \frac{1-3}{0-2} = \frac{-2}{-2} = 1$$

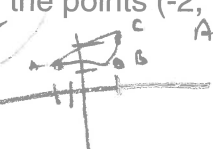
$$m_{AC} = \frac{0+1}{1-2} = \frac{1}{-1} = -1$$

$$1 \cdot -1 = -1 \quad \therefore \overline{AB} \perp \overline{AC}$$

$$\overline{AB} = \sqrt{(0-2)^2 + (1-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

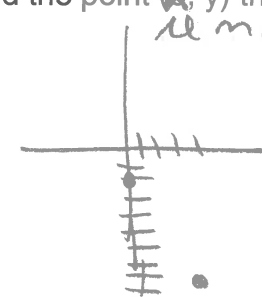
$$\begin{aligned} \overline{AC} &= \sqrt{(2-1)^2 + (3-(-1))^2} \\ &= \sqrt{1+16} \\ &= \sqrt{17} \end{aligned}$$

- 13) Find the length of the hypotenuse of the right triangle whose vertices are given by the points (-2, 1), (1, 1) and (1, 2).



$$\begin{aligned} \overline{AC} &= \sqrt{(-2-1)^2 + (1-2)^2} \\ &= \sqrt{(-3)^2 + (-1)^2} \\ &= \sqrt{9+1} = \sqrt{10} \end{aligned}$$

- 14) Find the point (x, y) that is equidistant from (4, -9) and (0, -2).

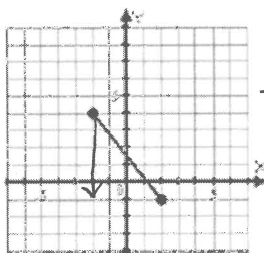


$$\begin{aligned} \left(\frac{4+0}{2}, \frac{-9-2}{2}\right) \\ \left(\frac{4}{2}, \frac{-11}{2}\right) \\ \left(2, -\frac{11}{2}\right) \end{aligned}$$

SLOPE

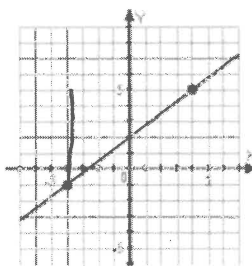
15) Find slopes of the following (use $\frac{\text{rise}}{\text{run}}$)

a.



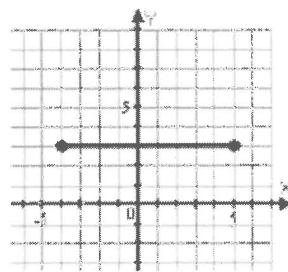
$$-\frac{5}{4}$$

b.



$$\frac{6}{2} = 3$$

c.

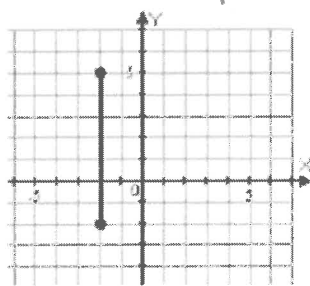


$$0$$

0 in numerator

0 in denominator

undefined



The slope of graph (a) should be negative because it's rising to the left. (falling to the right)

The slope of graph (b) should be positive because it's rising to the right.

The slope of graph (c) should be 0 because it's horizontal.

The slope of graph (d) should be undefined because it's vertical.

16) The total amount that Thomas earns in a day from his summer job at a gas station is dependent on the number of hours he works. If he works for 8 hours, he earns \$52.

a) Create a table of values for this function (hours vs earnings).

hours	earnings (\$)
8	52
16	104
24	156

b) What is the slope of the line? What does the slope represent?

$$\frac{156 - 104}{24 - 16} = \frac{52}{8} = 6.50 \text{ \$/hr. earnings per hour}$$

c) If Tom worked for 12 hours in a day, what amount would he earn?

$$12(6.50) = \$78.00$$

17) You are building a wheelchair ramp that leads to a doorway 22 inches above the ground. The slope of the ramp must be $\frac{1}{12}$. Find the length of ground in inches that the ramp covers. (Can convert answer to feet by dividing by 12, as there are 12 inches in a foot.)



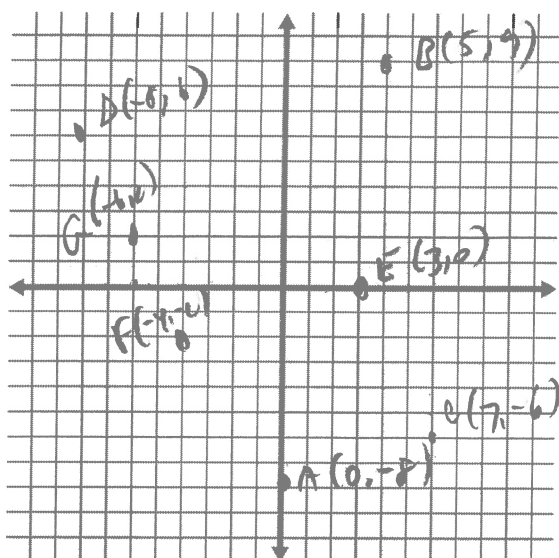
$$m = \frac{\text{rise}}{\text{run}}$$

$$\frac{1}{12} = \frac{22}{x}$$

$$x = 264$$

$$264 \text{ inches} = 22 \text{ feet}$$

18)



a. Given $A(0, -8)$, $B(5, 9)$, $C(7, -6)$, $D(-8, 6)$, $E(3, 0)$, $F(-4, -2)$, $G(-6, 2)$, label the points on the graph.

b. Find the slope of \overline{BE} and \overline{EG} . What does the slope indicate about these lines? How do you know?

$$m_{\overline{BE}} = \frac{9-0}{5-3} = \frac{9}{2} \quad m_{\overline{EG}} = \frac{0-2}{3+6} = \frac{-2}{9} \quad \text{They are } \perp. \quad (\text{slopes are neg reciprocals})$$

c. Find the slope of \overline{FE} and \overline{AC} . What does the slope indicate about these lines?

$$m_{\overline{FE}} = \frac{-2-0}{-4-3} = \frac{-2}{-7} = \frac{2}{7} \quad m_{\overline{AC}} = \frac{-8+6}{0-7} = \frac{-2}{-7} = \frac{2}{7} \quad \text{They are } \parallel. \quad (\text{slopes are } =)$$

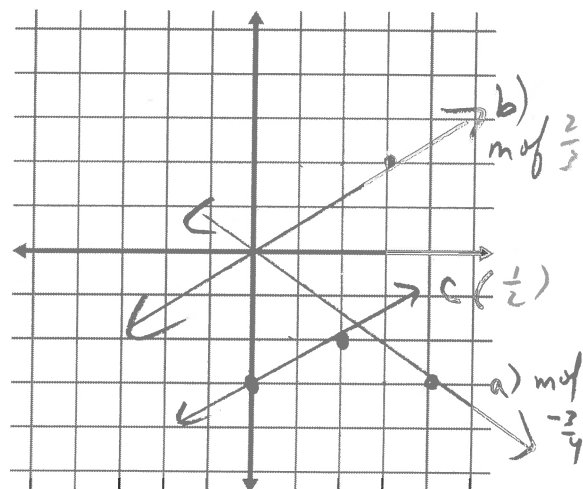
19) Graph lines with slope of: a) $-\frac{3}{4}$ b) $\frac{2}{3}$ c) 0.5

Which one(s) rise to the right? b + c

How do you know? positive

Which one(s) rise to the left? a

How do you know? negative



20) Are the three points $A(2, 3)$, $B(5, 6)$ and $C(0, -2)$ collinear (all on same line)?

$$m_{\overline{AB}} = \frac{6-3}{5-2} = \frac{3}{3} = 1 \quad m_{\overline{BC}} = \frac{6+2}{5-0} = \frac{8}{5} \quad \text{no, slopes are different.}$$

21a) What is the slope of the line $y = 9$?



21b) What is the slope of the line $x = -5$?



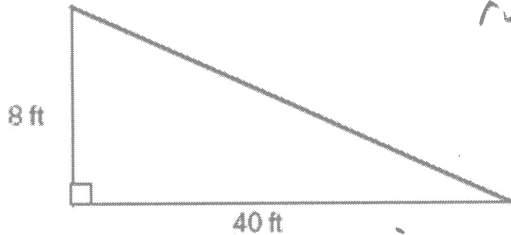
22) Find x if the line through the points (6, x) and (1, -5) has a slope of 2.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad 2 = \frac{x + 5}{5}$$

$$2 = \frac{x + 5}{6 - 1} \quad 10 = x + 5$$

$$\quad \quad \quad 5 = x$$

23) Use the rise and run measurements in the figure below to estimate the slope percent.



$$m = \frac{\text{rise}}{\text{run}} = \frac{8}{40} = 0.2$$

$$0.2 \times 100 = 20\%$$

24) A hill has a slope of 8 percent. The height of the hill is 15 feet. What is the horizontal distance?

$$8\% = 0.08$$

$$m = \frac{\text{rise}}{\text{run}} \quad 0.08 = \frac{15}{x}$$

$$0.08x = 15$$

$$x = \frac{15}{0.08}$$

$$x = 187.5 \text{ feet.}$$

25) Aurena is installing a pipe for drainage. The system requires a drop of 1.5 cm for every 2.5 cm of horizontal distance.

a) What is the slope of the pipe? $\frac{-1.5}{2.5} = -0.6$

b) How much drop will she need if the horizontal distance is 12m?

$$m = \frac{\text{rise}}{\text{run}} \Rightarrow -0.6 = \frac{x}{12} \Rightarrow -7.2 = x$$

(drop 7.2m)

26a) Write the slope of a line parallel to the line $y = \frac{-2}{3}x + 4$ $\frac{-2}{3}$

b) Write the slope of a line perpendicular to the line $y = \frac{-2}{3}x + 4$: $\frac{3}{2}$

c) What is the slope of the line perpendicular to the line $y = -2$? Undefined.

27) If you were given four points and were asked to find if a quadrilateral were a rectangle, write out the steps of your process to answer the question.

- ① Find if opposite sides equal using dist. formula.
- ② Find if 2 sides \perp using slope formula
(See if sides' slopes are negative reciprocals.)